Math 10A HW4 Solutions

(1) The distance between the point $(3,0)$ and (x,y) is given by:

$$d = \sqrt{(x-3)^2 + y^2}$$

But we have that $y = \sqrt{x}$ so

$$d = \sqrt{(x-3)^2 + x}$$

$$\Rightarrow d^2 = f(x) = (x-3)^2 + x$$

$$\Rightarrow f'(x) = 2(x-3) + 1 = 2x - 6 + 1$$

$$\Rightarrow f'(x) = 2x - 5 = 0$$

$$\Leftrightarrow x = 5/2 \text{ so } y = \sqrt{5/2}$$

(2) $y = 1 - 40x^3 - 3x^5$

$$\Rightarrow y' = -120x^2 - 15x^4$$

$$\Rightarrow y'' = -240x - 60x^3 = 0$$

$$-60x(4 + x^2) = 0$$

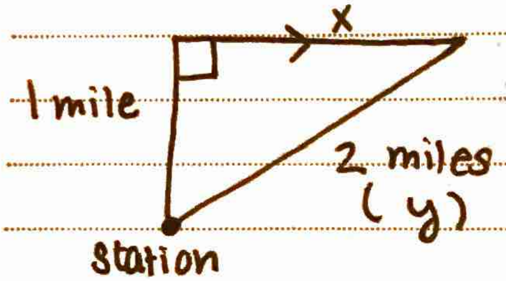
$$\Leftrightarrow x = 0 \text{ or } \cancel{x^2 = -4}$$

So the point on the curve at which the tangent line has the largest slope is $(0, y(0)) = (0, 1)$.

(3)

(4) Surface area (sphere) $= 4\pi r^2$
(i.e. $A(r) = 4\pi r^2$)
Want to find dA/dt for $r = 8$ cm so:
$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt} = 8\pi(8)(2) = 128\pi \text{ cm}^2/\text{min}$$

(5) We have the corresponding picture:



$$x^2 + 1^2 = x^2 + 1 = y^2$$
$$\Rightarrow 2x\frac{dx}{dt} = 2y\frac{dy}{dt}$$
$$\Rightarrow \cancel{2}(\sqrt{3})(500) = \cancel{2}(2)\frac{dy}{dt}$$
30-60-90 Δ
$$\Rightarrow \frac{dy}{dt} = 250\sqrt{3} \text{ miles/hr}$$

(6) $$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$
$$\Leftrightarrow \frac{1}{R} = \frac{R_2 + R_1}{R_1 R_2} \Leftrightarrow R = \frac{R_1 R_2}{R_1 + R_2}$$

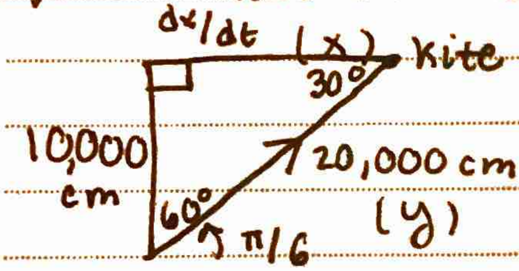
So we have:
$$\frac{dR}{dt} = \frac{(R_1+R_2)\left(R_1\frac{dR_2}{dt} + R_2\frac{dR_1}{dt}\right) - (R_1R_2)\left(\frac{dR_1}{dt} + \frac{dR_2}{dt}\right)}{(R_1+R_2)^2}$$

$$\Rightarrow \frac{dR}{dt} = \frac{(80+100)(80\cdot(0.2) + 100\cdot 0.3) - (8000)(0.1)}{(0.1)^2}$$

So $\frac{dR}{dt} = \frac{(180)[(-16)+30] - 800}{0.01} = \frac{1720}{0.01} = 172{,}000$ Ω/s

(increasing)

(7) We have the corresponding picture:



$$x^2 + (10{,}000)^2 = y^2$$

$$\cancel{2}x\frac{dx}{dt} = \cancel{2}y\frac{dy}{dt}$$

$$\Rightarrow \frac{dx}{dt} = \frac{(20{,}000)(3)}{(\sqrt{3}\cdot 10{,}000)}$$

$$\Rightarrow \frac{dx}{dt} = \frac{6}{\sqrt{3}} \text{ cm/s}$$

(8) Surface area (sphere) $= 4\pi r^2 = 4\pi\left(\frac{1}{2}\partial\right)^2$

$$A(r) = \pi\partial^2$$

Want to find $\frac{d\partial}{dt}$,

$$\frac{dA}{dt} = 2\pi\partial\frac{d\partial}{dt}$$

$$\Rightarrow -1 = 2\pi(10)\frac{d\partial}{dt}$$

$$\Leftrightarrow \frac{d\partial}{dt} = \frac{-1}{20\pi} \text{ cm/min}$$

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(9)

(a) $\displaystyle \lim_{x \to 3} \frac{(x-3)}{(x+3)(x-3)} = \lim_{x \to 3} \frac{1}{(x+3)} = \frac{1}{6}$

(b) $\displaystyle \lim_{x \to 0} \frac{\tan(3x)}{\sin(2x)} = \lim_{x \to 0} \frac{3\sec^2(3x)}{2\cos(2x)} = \frac{3}{2}$

(c) $\displaystyle \lim_{\theta \to \pi} \frac{1+\cos(\theta)}{1-\cos(\theta)} = 0$

(d) $\displaystyle \lim_{x \to \infty} \frac{x+x^2}{1-2x^2} = \lim_{x \to \infty} \frac{1+2x}{-4x} = \lim_{x \to \infty} \frac{2}{-4} = \frac{-1}{2}$

(e) $\displaystyle \lim_{x \to 1} \frac{\ln(x)}{x} = 0$

(f) $\displaystyle \lim_{x \to \infty} \frac{\ln(\sqrt{x})}{x^2} = \lim_{x \to \infty} \frac{1/\sqrt{x} \cdot \frac{1}{2}x^{-1/2}}{2x} = 0$

(g) $\displaystyle \lim_{t \to 1} \frac{t^8-1}{t^5-1} = \lim_{t \to 1} \frac{8t^7}{5t^4} = \frac{8}{5}$

(h) $\displaystyle \lim_{x \to 0} \frac{e^x-1-x}{x^2} = \lim_{x \to 0} \frac{e^x-1}{2x} = \lim_{x \to 0} \frac{e^x}{2} = \frac{1}{2}$

(i) $\lim_{x\to 0} \frac{\sin(x)-x}{x^3} = \lim_{x\to 0} \frac{\cos(x)-1}{3x^2}$

$= \lim_{x\to 0} \frac{-\sin(x)}{6x} = \lim_{x\to 0} \frac{-\cos(x)}{6} = \frac{-1}{6}$

(j) $\lim_{x\to 0} \frac{x-\sin x}{x-\tan x} = \lim_{x\to 0} \frac{1-\cos(x)}{1-\sec^2(x)}$

$= \lim_{x\to 0} \frac{\sin(x)}{-2\tan(x)\sec^2(x)}$

$= \lim_{x\to 0} \frac{\cancel{\sin(x)}\cos(x)}{-2\cancel{\sin(x)}\ ^1/\cos^2(x)}$

$= \lim_{x\to 0} \frac{\cos^3(x)}{-2} = \frac{-1}{2}$

(k)

$\lim_{x\to\infty} x\sin(\pi/x) = \lim_{x\to\infty} \frac{\sin(\pi/x)}{1/x} = \lim_{x\to\infty} \frac{\cos(\frac{\pi}{x})(\frac{-\pi}{x^2})}{-1/x^2}$

$= \lim_{x\to\infty} +\pi\cos\left(\frac{\pi}{x}\right) = \pi \lim_{x\to\infty} \cos(\pi/x) = \pi$

(l) $\lim_{x\to 0} \csc(x) - \cot(x) = \lim_{x\to 0} \frac{1}{\sin x} - \frac{\cos x}{\sin x}$

$= \lim_{x\to 0} \frac{1-\cos(x)}{\sin(x)} = \lim_{x\to 0} \frac{\sin(x)}{\cos(x)} = 0$

(m) $\lim_{x\to\infty} x - \ln(x) = \lim_{x\to\infty} \ln(e^x) - \ln(x)$

$= \lim_{x\to\infty} \ln\left(\frac{e^x}{x}\right) = \ln\left(\lim_{x\to\infty} \frac{e^x}{x}\right)$

$= \ln\left(\lim_{x\to\infty} e^x\right) = \infty$

(n) $\lim_{x\to-\infty} x\ln(1-\frac{1}{x}) = \lim_{x\to-\infty} \frac{\ln(1-\frac{1}{x})}{1/x}$

$= \lim_{x\to-\infty} \frac{(\cancel{x^{-2}})/(1-1/x)}{-\cancel{x^{-2}}}$

$= \lim_{x\to-\infty} -\left(1-\frac{1}{x}\right) = -1$

(o) $\lim_{x\to\infty} x^{e^{-x}}$

Let $y = x^{e^{-x}}$ then $\ln(y) = e^{-x}\ln(x)$.

$\lim_{x\to\infty} \ln(y) = \lim_{x\to\infty} \frac{\ln(x)}{e^x} = \lim_{x\to\infty} \frac{1}{xe^x} = 0$

$\Rightarrow \lim_{x\to\infty} e^{\ln(y)} = 1$

(10) False!

Let $f(x) = x^2/2$ so $f'(x) = x$

$g(x) = -\cos(x)$ so $g'(x) = \sin(x)$

with $c = 0$.